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## **DESIGN EMBEDDING: REPRESENTATION LEARNING OF DESIGN THINKING TO CLUSTER DESIGN BEHAVIORS**

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### **ABSTRACT**

*Design thinking is essential and the core of the design process as it helps to achieve the design goal by governing design decisions. Therefore, understanding design thinking is vital for improving design methods, tools and theories. However, interpreting design thinking is challenging because it is a cognitive process that is hidden and intangible. In this paper, we represent design thinking as an intermediate layer between the thought process and design behaviors. To do so, this paper first identifies five design behaviors based on the current design theories. These behaviors include action behavior; one-step sequential behavior; contextual behavior; long-term sequential behavior; and reflective thinking behavior. Next, we develop computational methods to characterize each of the design behaviors. Particularly, we use design action preference distribution, first-order Markov chain model, Doc2Vec, bi-directional LSTM autoencoder, and time gap distribution to characterize the design behaviors. The characterization of the design behaviors through embedding techniques is essentially the latent representation of the design thinking, which is referred to as design embeddings. After obtaining the embedding, an X-mean clustering algorithm is applied on each of the embeddings to group the designers. The approach is applied to data collected from a high school solar system design challenge. The clustering results show that designers follow several design patterns according to the corresponding behavior, which corroborates design embedding effectiveness. Successful implementation of this method to identify design embedding can be useful in other design research, such as inferring design decisions, predicting design performance, and identifying design actions identification.*

**Keywords:** Design thinking, design embedding, design cognition, deep learning.

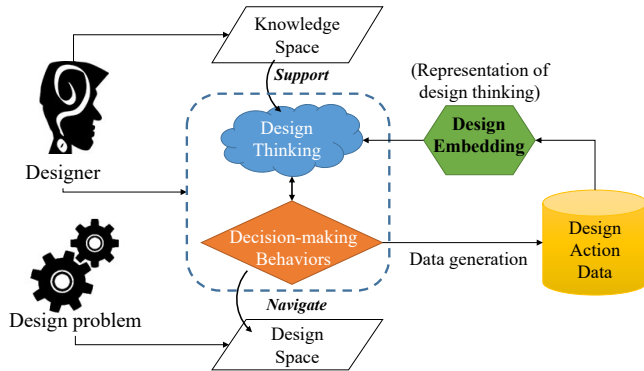
### **1. INTRODUCTION**

Design thinking governs how designers apply the design principles to generate, evaluate, and represent concepts to meet stated goals [1] [2]. In the context of engineering design, design thinking refers primarily to the exploration (i.e., divergent thinking) and exploitation (i.e., convergent thinking) iterations in search of design solutions [3]. More generally speaking, design thinking is designers' cognitive activities during a design process in which their decision-making strategies and behaviors are guided by their design thinking, and their corresponding actions are reflected through the design task. Therefore, design thinking works as a bridge that connects designers' knowledge space and design space [4], as shown in Figure 1. A deeper understanding of design thinking is vital for advancing design theories, methods, and tools.

However, understanding and interpreting design thinking are challenging because it is intangible and takes place in the human brain [ref]. During a design task, different designers may adopt different design strategies. Thus, the design behaviors that reflect their design thinking are different too [Dinal et al.]. This is particularly true in complex systems design, where the problem often involves various design variables and constraints. For example, in one of our previous studies, several design patterns were identified in the same solar system design task by studying designers' one-step sequential decision-making behaviors [5]. In order to fundamentally understand design thinking, various empirical studies have been conducted based on different methodologies, such as protocol methods, controlled experiments, psychological tests, and neuroscientific measurement, such as functional magnetic resonance imaging (fMRI) [2]. While existing studies have leveraged the advancement in machine learning and data mining techniques in

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**FIGURE 1: THE CONNECTION BETWEEN DESIGN THINKING AND DESIGN EMBEDDING**

discovering behavioral patterns in design from which we could draw insights and inferences about their design thinking [2], little research was done on understanding the latent representations of design thinking. We define the representation of design thinking as an intermediate layer between human designer's mental processes (i.e., the thought process) and their behaviors (i.e., actions). Our hypothesis is that the design thinking representation is potentially an essential and effective pathway to the empirical studies of designers' thinking.

Now suppose design thinking is an abstraction and mapping of design behaviors at a high-dimensional space, then the understanding of design thinking must not be acquired from a single behavioral type. If multiple-dimensional design behaviors and the corresponding patterns are identifiable, then a series of questions are, would the representation of design thinking extracted from different design behaviors be different? How does the representation of design thinking obtained from each dimension of the design behavior look like? What are the ways, particularly computational methods, to extract such representations?

As the first attempt to answer these questions, on the one hand, we identify five different design behaviors, including short-term sequential behavior, long-term sequential behavior, contextual behavior, reflective thinking, and design action preference, based on current research on design theories. Each of these design behaviors is elaborated in Section 2. On the other hand, we explore the possibility of using embedding techniques from machine learning to transform high-dimensional design action data into low-dimensional embeddings, referred to as *design embeddings*, for the latent representation of design thinking. In the literature of machine learning, an *embedding* is a low-dimensional vector representation of high-dimensional data [ref]. Embedding maps discrete, categorical variables to a vector of continuous numbers. Figure 1(b) illustrates the connections among design thinking, design embeddings, and design behaviors.

In this study, our assumption is that design thinking is reflected by design behaviors in multiple dimensions. Therefore, by abstracting and extracting the latent representation of design behavioral data in the transformed dimension via embedding techniques, design thinking can be better characterized.

Particularly, we develop an approach that applies different embedding techniques to learn design thinking representations from designers' action data. The scope of this study is focused on computer-aided design (CAD) for the ease of data collection. However, the approach is applicable in any design context as long as the design action data can be collected. This approach is demonstrated using the data collected from a high-school student CAD challenge where participants are asked to solarize their school with the required energy yield and payback period (see Section 4.2 for detail).

The remaining of the paper is as follows. In Section 2, we present the literature review on design thinking studies and summarize the common representation of design thinking in various data types. In Section 3, we present the overall research approach and discuss the technical background regarding the different embedding techniques adopted in this study. In Section 4, a case study on the solar system design challenge is presented. Also, we discuss the experiment details and data collection method in this section. The results are presented and discussed in Section 5. Finally, in Section 6, we wrap up this paper by drawing conclusions and insights as well as a summary of limitations which opens up the opportunities for our future work on the topic of design thinking representation.

## 2. LITERATURE REVIEW

### 2.1 Representation of design thinking

Extensive studies have been conducted to study design thinking. These studies adopted various ways to represent design thinking, such as by using cognitive study (e.g., protocol study, controlled experiment), physiological measurement (e.g., eye tracking, heart rate, electrocardiography (ECG)), neurological signals (e.g., electroencephalogram (EEG), functional magnetic resonance imaging (fMRI)) [6]. In protocol and controlled study, design data are encoded by ontological design model (i.e., function-behavior-design (FBS) design process model), which are collected from protocol study or controlled experiment [7]. These design data are typically designers' performed actions [8] and are further encoded to a deeper understanding of design thinking [9]. The encoded design data is analyzed by different computational methods in order to represent design thinking. For example, the first-order Markov chain model representing one-step sequential decision-making behavior is utilized to study design pattern [5], [10], hidden Markov model to identify hidden design states [8], long short-term memory unit (LSTM) to predict future design process stage [11]. In some studies, sketch data are collected besides the verbal and design action data [12]. Sketching is further encoded using different sketch coding methods (e.g., C-sketch method [13]) to represent design thinking.

Design thinking is also studied using various physiological measures such as eye-tracking, electrocardiogram (ECG), and facial recognition. In the eye-tracking method, eye-tracking devices and software capture designers' eye movement and provide gaze point and heat maps of areas of interest [14]. The heat maps and gaze points represent designers' thinking. These

data has been used to analyze design creativity [15], to study how designers analyze the functionality of a design object [16]. This method mainly analyzes how much attention designers put on the area of a specific design object. Using ECG, heart rate variability (HRV) signals can be recorded and connected to mental stress [17]. HRV is measured during the different design segments, and the corresponding mental stress is measured. Different designers show different patterns of stress according to their design thinking [ref].

Data collected from neurological studies try to connect design thinking and brain activity. The two most popular methods for neurological studies are EEG and fMRI. While EEG measures neural activity via the identification of electrical current, fMRI measures brain activity by the brain's blood flow using a magnetic field [6]. Typically, EEG data is transformed in order to understand different aspects of design thinking, such as the transformed power of the sensor measurement for identifying the differences between problem-solving design and open-ended design [18]. From the EEG data, the power spectral density of brain waves is measured, and the correlation between design activity and brain waves is analyzed [19]. Data from fMRI are images of the brain at cross-sections that provide visual reasoning, such as brain activation patterns during design ideation [20]. Recent studies conducted by neurocognition scientists indicated that when designers engaged in divergent thinking, different cognitive domains were activated with the tasks that require analysis during the engineering concept generation [21]. Design neurocognition researchers also have successfully encapsulated the cognitive functioning behind engineering design [18]. This empirical research confirmed that design thinking is not merely an abstract construct. However, the external design behavior regulated by different cognitive processes involved during the search of design solutions requires further investigation through the study of design actions [22].

## 2.2 Behaviors in the design process

The design process involved various behaviors, among which sequential behavior is considered as an integral part [23] and a natural feature of design competency [24]. Many types of research have been conducted to study designers' sequential behavior using the Markov chain model. Typically, the first-order Markov chain model is utilized to study designer transition behavior or one-step sequential behavior. This behavioral study is used to identify design patterns [10], [5], to study designers' sequential learning process [25]. The Second-order and higher-order Markov chain model represents short-term sequential behavior. Several studies utilize those models to study the design process. For example, to compare the design process between two design domains: architects and software designers, second-order MC has been implemented [26]. The Higher-order Markov model is adopted in an agent-based modeling framework to study the effect of memory on sequential behaviors [27]. The hidden Markov model and Deep learning-based model are used to model designers' long-term sequential behavior. For example, in our previous study [5], by using the long-short term memory (LSTM) unit, it is identified that designers use both long-term

and short-term memory in the design process. In the same study, the hidden Markov model (HMM) is used to predict long design sequences. HMM is also used to extract design strategies to create a computer agent that can solve truss design problems [28].

In addition to different sequential behaviors, studies have also been conducted on other types of behaviors, such as reflective thinking. Reflective design thinking is a conscious mental activity that examines designers' design actions, decisions, and inner selves throughout a design process [29]. Though the study of reflective thinking is a growing trend, very few studies have been conducted on design reflection [30]. Goldstein et al. [30] use designers' electronic notepad and pre-test and post-test to study designer reflective thinking and found that moderately reflective students understood design activities better than those with high or low reflectivity. Even though many studies on design behaviors have been conducted, most of them focus a particular design behavior at a time. However, design thinking is not merely a particular design behavior; rather, it is an abstraction of design behaviors from multiple dimensions. Therefore, to a deeper understanding of design thinking, a study on different design behaviors is needed.

## 3. TECHNICAL BACKGROUND AND RESEARCH APPROACH

In this section, first, we briefly introduce the research approach adopted in this study. Next, we present the technical background for different embedding techniques.

### 3.1 Theoretical background

One of the major contributions of this study is to identify five design behaviors for studying the design thinking representation. Therefore, before describing the overall research approach, we would like to present the rationale of choosing the five behaviors, including action behavior, one-step sequential behavior, contextual behavior, long-term sequential behavior, and reflective thinking behavior.

The one-step sequential behavior, contextual behavior, and long-term sequential behavior are selected based on the mental iteration model [31]. Design is a goal-directed problem-solving process and can be modeled as an iterative and sequential decision-making process. Jin and Chuslip [31] proposed a cognitive model to describe the mental iteration during design. According to that model, in every design process, several cognitive activities occur, such as generate, compose, evaluate, etc. Also, different iteration loops are embedded in the design process. These loops collectively generate a global loop. Besides the global loop, each cognitive activity defines a local loop. In complex systems design problems, these loops frequently occur as designers go back and forth iteratively between different stages to search the design space and take different design actions to accomplish required design tasks. Therefore, in this study, we propose to use one-step sequential behavior and contextual behavior (short-term behaviors) to capture the local loop design behavioral patterns and use long-term sequential behaviors to capture the global loop iterative patterns.

Next, we consider reflective thinking. The core of reflective thinking is metacognition and self-monitoring, which help designers to reflect experience and knowledge in their actions as well as provide feedback to improve the design process [32]. In the design process, designers take various modes of reflective thinking. For example, some designers use a bigger picture (take a longer time to think) while others use a micro-scoping view (take a shorter time to think). Reflective thinking behavior enables designers to scrutinize their thinking, behavior, design process and thus produce higher quality designs [33][34]. Therefore, understanding and computationally modeling designer reflective thinking are important.

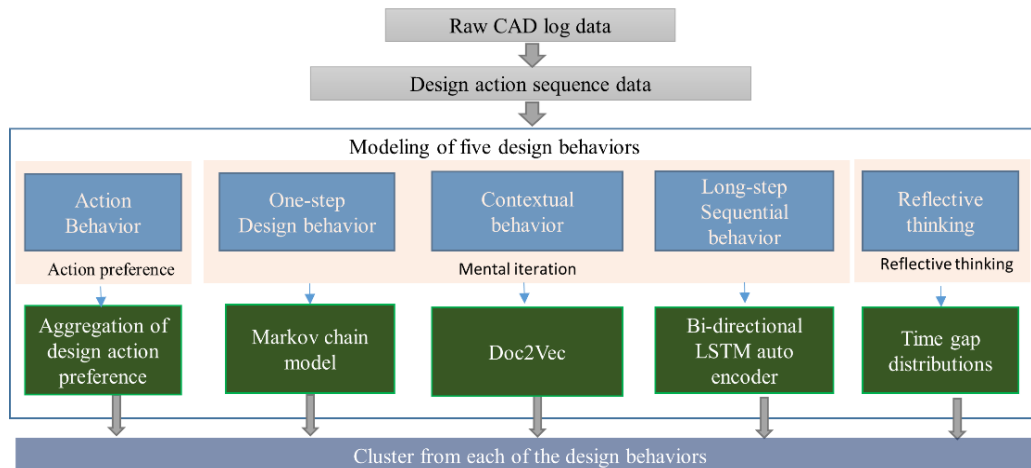
Lastly, we study designers' action preferences based on the designers' action preferences. It depicts how frequently a designer uses different types of design actions (i.e., the distribution of design actions) during the design process. In total, five different design behaviors are adopted from three dimensions – mental iteration, reflective thinking, and design action preferences. We envision that modeling the design behaviors from multiple dimensions can help better understand design thinking.

### 3.2 Research approach

The overall approach starts with collecting the raw design action data from different sources such as CAD loggers, design documents, etc. This raw design action data contains design actions, design-related artifacts, design parameters, etc. After collecting the design action data, to computationally model these five design behaviors, we adopted five different techniques. We use aggregated design action preference to model design action preferences, Markov model to model one-step sequential behavior, Doc2Vec to model contextual behavior, Bi-directional LSTM autoencoder to understand long-term sequential behavior, and time-gap distribution analysis for understanding reflective thinking. To explain the overall process, suppose a designer's sequence of design actions  $[a_1, a_2, a_3 \dots \dots a_N]$  which has time stamp associated with it  $[t_1, t_2, t_3 \dots \dots t_N]$ .

Before analyzing the aggregated design action preference and one-step sequential behavior using the Markov chain, we apply an ontological design process model (e.g., the function-behavior-structure model) action sequences, which consist of several design stages to define the design process. By applying the design process model, we will obtain a sequence of design process stages  $[p_1, p_2, p_3 \dots \dots p_N]$ . With this operation, we can reduce the dimensionality of the design sequence data. This is similar to an embedding (latent space representation), which can help interpret designers' thinking and decision-making processes. To elicit designers' action preferences, we count the total number of each design process stage that certain actions fall into and plot the resulting distribution for every designer. All designers' action preference vectors are then concatenated to form an aggregated matrix representing design action preferences. To understand designers' one-step sequential behavior, we apply the first-order Markov chain to the design process stages. For every designers' sequence, we compute the transition probability matrix for every designers' action sequence based on the Markov chain model. This transition probability matrix can be vectorized, which quantifies the features of the one-step sequential behavior. Having  $N$  design process stages of a design process model, for a particular designer, we get  $N \times 1$  vector from action preference and  $N \times N$  transition probability from Markov chain model. The transition probability can be converted into  $N^2 \times 1$  vector. For  $n$  designers, respective  $N \times n$  and  $N^2 \times n$  matrix can be formed. These two matrices represent the aggregated action behaviors and one-step sequential behavior, respectively.

To understand designers' contextual behavior and long-term sequential behavior, we apply the Doc2Vec [35] and bi-directional LSTM auto-encoder [36], respectively, on the design action sequence. In Doc2Vec and bi-directional LSTM architecture, both attempt to predict an element from the input sequence. Doc2Vec or paragraph vectors support this prediction process by training paragraph vectors as auxiliary information. We will get an embedding matrix from each of these methods. As the embedding matrix is already a representation of the



**FIGURE 2: THE RESEARCH APPROACH FOR STUDYING DESIGN THINKING BASED ON FIVE DESIGN BEHAVIORS**

relationship among design actions, the data transformation from design action to design process stage using an ontological design process model is not needed in these two methods. It is mention-worthy that the size of the embedding matrix is user-defined. For example, with the embedding size of  $M$ , and for  $n$  designers' sequence, we get  $M \times n$  dimensional matrix from each of the methods.

To understand the designers' reflective thinking, we utilize the time-gap distribution analysis. Particularly, we consider the time gap corresponding to each of the design actions performed by a designer. For example, for the sequence, the time gaps are  $[0, \{t_2 - t_1\}, \{t_3 - t_2\} \dots \dots \{t_n - t_{n-1}\}]$ . The distribution of this time gap essentially carries the reflective behavior. From each of the designers' time gap distribution, we obtain several features, such as the distribution name and distribution parameters. For a particular designer, we use these features to create a vector,  $P = [Dist\ name, D_1, D_2, D_3]$ , where *Dist name* indicates the distribution name (a categorical variable) and  $D_1, D_2, D_3$  are the distribution parameters. It is noted that the parameter number can be varied based on the type of the distribution. Assuming there are  $L$  parameters, for  $n$  designers, we obtain an  $L \times n$  matrix. This matrix will be the feature representation of designers' reflective design behaviors.

Based on these five models, we can obtain a behavioral matrix (i.e., the design embeddings) from each of these five corresponding design behavior. Then, we implement a clustering method, i.e., X-mean cluster [37], on each behavioral matrix to identify the designers who have similar design behavioral patterns. Figure 2 depicts a schematic diagram of the research approaches.

### 3.3 Doc2Vec

Doc2Vec or paragraph vector uses a neural network approach to create a fixed-length vector representation of variable length sequences such as sentences, paragraphs, or designers' action sequences. Doc2Vec is based on Word2Vec, where it attempts to predict an element in a sequence from its surrounding element [35]. Doc2Vec or paragraph vectors support this prediction process by training paragraph vectors as auxiliary information. Given a sequence  $w_1, w_2, w_3, \dots, w_T$ , to predict the context element  $w_t$ , the objective of the Word2vec is to maximize the log probability.

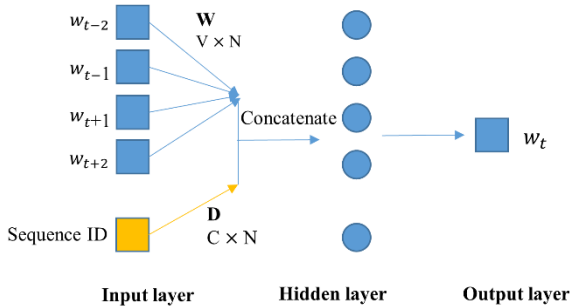


FIGURE 3: Doc2Vec

$$\frac{1}{T} \sum_{t=k}^{T-k} \log p(w_t | w_{t-k}, \dots, w_{t+k}) \quad (1)$$

The prediction task is typically done by a neural network architecture where the prediction is made through a multiclass classifier such as softmax [38]. This process can be expressed as follows:

$$p(w_t | w_{t-k}, \dots, w_{t+k}) = \frac{e^{y w_t}}{\sum_i e^{y_i}} \quad (2)$$

$$y = b + \mathbf{U} \mathbf{h}(w_{t-k}, \dots, w_{t+k}; \mathbf{W}) \quad (3)$$

, where equation 2 represents the softmax function. Each of  $y_i$  is the log probability for each output element  $i$ .  $\mathbf{U}, b$  are the parameter of neural networks.  $\mathbf{h}$  is constructed by a concatenation of vectors extracted from  $\mathbf{W}$ .

In Doc2Vec or paragraph vector framework, every sequence is associated with a unique vector, which is represented by a matrix  $\mathbf{D}$  (for all sequences, it creates a matrix). Every element of the sequence is also mapped to a unique vector which is represented as  $\mathbf{W}$  in Figure 3. The matrix  $\mathbf{D}$  and  $\mathbf{W}$  are concatenated and used in Equation (1) in place of  $\mathbf{h}$ .

### 3.4 Bi-directional LSTM auto-encoder

The aim of using an auto-encoder (AE) is to learn a compressed, distributed representation of a data set. It is a neural network model that captures the most salient features of the input data [39]. The basic AE consists of only one hidden layer, and the target value is set equal to the input value. The training of the AE is done in two phases: encoding and decoding. In the encoding phase, input data are mapped into the hidden layer, and in the decoding process, the input data are reconstructed from the hidden layer representation. Given an input dataset  $X = x_1, x_2, x_3, \dots, x_n$ , the two phases can be expressed as follows:

$$\mathbf{h}(x) = f(\mathbf{W}_1 x + b_1) \quad (4)$$

$$\hat{x} = g(\mathbf{W}_2 \mathbf{h}(x) + b_2) \quad (5)$$

, where,  $\mathbf{h}(x)$  represents the hidden representations of the input vector  $x$ , and  $\hat{x}$  is the decoder vector of the output layer.  $f$  is the encoding function, while  $g$  is the decoding function.  $\mathbf{W}_1$  and  $\mathbf{W}_2$  are the weight matrix of the encoder and decoder, respectively.

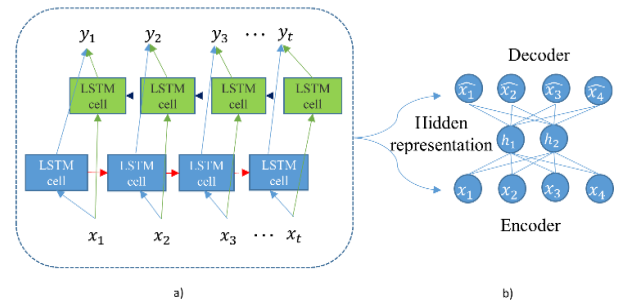
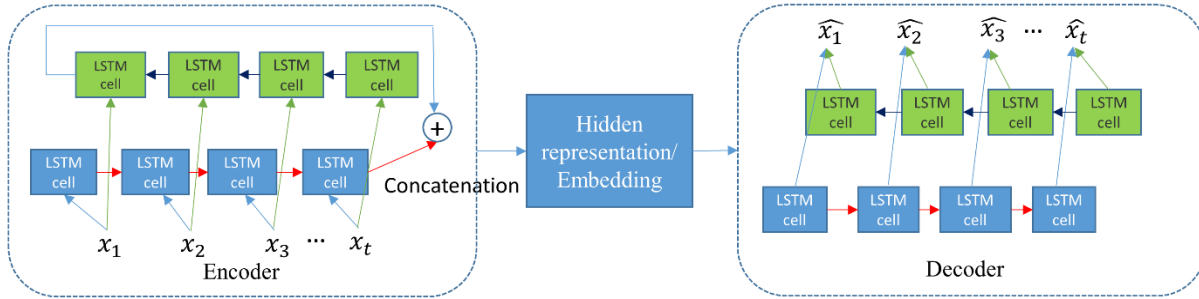


FIGURE 4 a): BI-DIRECTIONAL LSTM b) BASIC AUTO-ENCODER





**FIGURE 5: BI-DIRECTIONAL LSTM AUTO-ENCODER**

$b_1$  and  $b_2$  are the bias vector in each phase, respectively. A schematic diagram of the auto-encoder is shown in figure 4 (b). LSTM is an upgraded variation of the recurrent neural network (RNN) [40], which is basically a recursive neural network used for sequential data. LSTM uses a gating mechanism that solves several flaws of the RNN (i.e., vanishing gradient problem, long-term dependency, etc.). A detail of the LSTM network is described in our previous work [11]. In this study, we leverage bidirectional LSTM in the auto-encoder architecture. Compared to the basic LSTM model, bidirectional LSTM consists of two groups of hidden layers. One layer for input sequence in the forward direction and the other layer for input sequence in the backward direction. These two hidden layers do not interact with each other, and their output is concatenated to the final output layer. The mathematical equations for the bidirectional LSTM are the same as basic LSTM, except there are two hidden states at  $t^{th}$  time steps:  $\vec{h}$  (forward process) and  $\overleftarrow{h}$  (backward process). These two hidden states are concatenated for the final output

$$H = \text{concat}(\vec{h}, \overleftarrow{h}) \quad (6)$$

In the AE architecture, bi-directional LSTM is replaced with the feed-forward neural network. A schematic diagram of the bi-directional LSTM autoencoder is shown in figure 5.

#### 4. CLUSTERING DESIGN BEHAVIORS IN SOLAR ENERGY SYSTEM DESIGN - A CASE STUDY

In this section, we first provide an introduction to the design challenge and the data collection method.

##### 4.1 Design procedure

The study was implemented in a suburban high school in the North-eastern US. The participants are 113 students from seven 9th-grade classes on the course of the science of energy. These students barely had design experience before the project. During the six-day project, students worked with an open-source CAD tool named Energy3D [41] individually and sought help from teachers when needed. Specifically, the project started with a day of Energy3D tutorial and followed by three days of conceptual learning, in which students interacted with simulations to understand five solar concepts (e.g., the Sun's path over a year) and how these concepts affect solar-energy acceptance. Then

students worked to solve an authentic design challenge for two days to apply knowledge to practice and develop design skills.

##### 4.2 Design problem

The five solar concepts are the Sun's path, the projection effect, the effect of the air mass, the effect of weather, and solar radiation pathways. These concepts are tightly related to the design challenge and were selected by domain experts in the research team. Individual simulations and exercises were provided to students to learn each concept. The design task was customized to the students with their school as the context. The challenge was named Solarize your school and set as asking for bids to power their school with green energy. Mainly, a 3D model of their school was provided. Students could install solar panels on the school building roof to generate no less than 400,000 kWh of electricity per year while the payback period was less than ten years. We provide three different solar panel models from which designers can choose any one of them for the design (see figure 6). This design challenge required students to balance several factors such as panel costs, solar panel orientation, tile angle, and avoiding shadows while aiming for the goal.

##### 4.3 Data collection and data processing

Energy3D collects the continuous flow of design logs which includes design actions, time steps, design parameters, and simulation results. Although initially, we collect 113 designers'



**FIGURE 6: AN EXAMPLE OF THE SOLARIZE YOUR SCHOOL DESIGN**

data, after analyzing their design, we realize that several students did not follow the design requirements (i.e., choose a different one other than the provided solar panels). For a fair comparison, we only consider the designs that follow the design constraints, and in this way, we identified 39 valid designs. An example of a line of design action log is shown below:

```
{"Timestamp": "2019-10-22 08:34:26", "Project": "Stoughton High School", "File": "stoughton-high-school-ma.ng3", "Change Tilt Angle for All Racks": {"New Value": -1.0}}
```

In this study, we only collect actions that are related to design, such as adding a component or modifying a component, etc. However, we ignore the camera-related action such as “zoom in,” “zoom out,” and “camera” because it does not affect the design directly. After removing the irrelevant design actions, there are 60 unique design actions. Then for action behavior and one-step sequential behavior, we develop a coding scheme based on the FBS model to transcribe the design actions data into design process stages. The coding scheme shown in table 1 is used to categorize each design actions into one of the seven design process stages, including Formulation (F), Analysis (A), Synthesis (S), Evaluation (E), Reformulation 1(R1), Reformulation 2 (R2) and Reformulation 3 (R3). The detail of the transformation process is described in our prior work [42].

**TABLE 1: CODING SCHEME BASED ON FBS DESIGN PROCESS**

Design process-stages	Design action
Formulation	Add any component
Analysis	Analysis of annual net energy
Synthesis	Edit any component
Evaluation	Cost analysis
Reformulation 1	Remove structure
Reformulation 2	Remove solar device
Reformulation 3	Remove other components

## 5. RESULT AND DISCUSSION

### 5.1 Result

In this section, we present the result obtained from different design behaviors, particularly action behavior, one-step sequential behavior, contextual behavior, long-term sequential behavior, and reflective thinking. The behaviors are represented as embedding and clustered using the X-mean clustering method. To compare the performance of designers of each of the clusters, we developed a metric using the design performance data. The metric is as follows:

$$Metric = \frac{P_R \times B \times E_0}{P_0 \times C \times E_R}$$

Here,

$P_R$  = required payback period

$B$  = budget

$E_0$  = Obtained energy output

$P_0$  = Obtained payback period

$C$  = Cost

$E_R$  = required energy output

Action behavior is obtained from the aggregation of design process stages preference. We apply the FBS design process model to convert the design actions to the design process stages. Applying the FBS design process model, we get seven design process stages. For each of the designers, we get a  $7 \times 1$  vector, and for 39 designers, we get a  $7 \times 39$  action behavior matrix. By applying X-mean clustering on the action behavior matrix, three clusters are found. Cluster 3 includes ten designers who achieve the highest mean performance of 1.325 with a standard deviation of 0.40, while cluster 1 achieves the lowest performance with 1.2077 (standard deviation 0.408). Cluster 2 contains 13 designers with a mean performance of 1.25 (standard deviation 0.64). Analysis of variance (ANOVA) indicates the difference in the cluster's design performance is not significant (p-value is 0.708).

We quantify the one-step sequential behavior using the first-order Markov chain model. Particularly, the transition

**TABLE 2: CLUSTER OF ONE-STEP SEQUENTIAL BEHAVIOR**

	Cluster 1	Cluster 2
0	P1L10	P1L12
1	P1L14	P1L13
2	P1L17	P1L20
3	P1L18	P1L3
4	P2L10	P1L5
5	P2L12	P2L11
6	P2L13	P2L2
7	P2L14	P4L1
8	P2L16	P4L10
9	P2L17	P4L25
10	P2L7	P4L28
11	P3L3	P4L32
12	P4L11	P4L5
13	P4L26	P6L12
14	P4L27	P6L17
15	P4L9	P6L18
16	P6L1	P6L3
17	P6L14	
18	P6L15	
19	P6L19	
20	P6L4	
21	P6L6	
Mean of design performance	1.255333	1.247206
STD of design performance	0.277998	0.647655

probability matrix obtained from the first-order Markov model is characterized as the one-step sequential behavior. Like the previous method, before applying the Markov model, we apply the FBS design process model to transform the design action into the design process stage. We obtain a  $7 \times 7$  transition probability matrix for seven design process stages and then flatten the matrix to obtain a  $49 \times 1$  vector. After obtaining 39 designers' transition probability matrix, they are converted to a  $39 \times 49$  matrix that captures the one-step design behavior, from which X-means clustering is applied. By clustering one-step sequential behavior, we identify two clusters. In this method, the performance obtained from the clusters is similar. Cluster 1 contains 22 designers with a mean performance of 1.253 (standard deviation 0.27), while cluster 2 achieves a 1.247 mean performance score with a standard deviation of 1.325. However, the t-test indicates that there are no significant differences among the performance of the cluster groups (p-value 0.27). Table 2 shows the result of clustering one-step sequential behavior.

Using Doc2Vec, we obtain design embedding that represents the designers' contextual behavior or short-term behavior. In this method, we directly use the design actions since Doc2Vec provides an embedding matrix of a particular size representing the relationship among design actions. There are several hyperparameters that need to be tuned and selected for the Doc2Vec model. For example, in this study, we choose the embedding size for Doc2Vec as 100. Additionally, we choose the context window size as 5. It is mention-worthy that the settings of the hyperparameter are user-defined. With these settings, for 39 designers, we obtain a  $39 \times 100$  embedding matrix. We apply the X-means cluster on the obtained embedding matrix and get two clusters. The first cluster contains 30 designers with a mean performance of 1.22 and a standard deviation of 0.48. The second cluster contains nine designers with a mean performance of 1.337 and a standard deviation of 0.43. However, the t-test between the clusters indicates that their difference is not significantly higher than the others (p-value 0.27).

We obtain design embedding for the long-term sequential by utilizing the Bi-directional LSTM autoencoder. In this architecture, in both the encoder and decoder part, we use a bi-directional LSTM layer with a size of 128. Therefore, the embedding size from the LSTM autoencoder is 256, and with all the designers, we obtain a  $39 \times 256$  matrix. By clustering the embedding matrix, we get three clusters. Table 3 shows the clustering result of the LSTM autoencoder. Cluster 1 contains 24 designers with a mean performance of 1.14 (standard deviation .37), while cluster 3 has only three designers with a mean performance of 1.35 (standard deviation 0.58). Cluster 2 contains 12 designers with a mean performance of 1.44 (standard deviation 0.55). According to the ANOVA test, the difference among the clusters is not significant (p-value 0.7).

Finally, to obtain the embedding from reflective thinking, we get the designers' time gap distribution parameters. In this study, we consider the time gap as the time before taking a design action. Also, it is mention-worthy that we consider the time gap between 0s to 300s. During design, some designers may wander

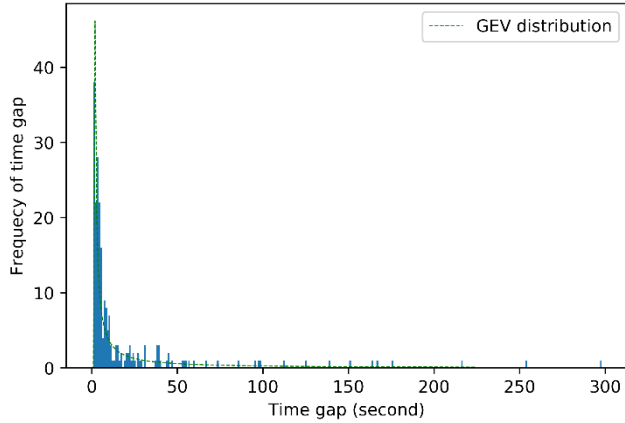
**TABLE 3: CLUSTER OF LONG-TERM SEQUENTIAL BEHAVIOR**

	Cluster 1	Cluster 2	Cluster 3
<b>0</b>	P1L10	P1L12	P1L3
<b>1</b>	P1L13	P2L14	P6L1
<b>2</b>	P1L14	P2L16	P6L18
<b>3</b>	P1L17	P2L2	
<b>4</b>	P1L18	P2L7	
<b>5</b>	P1L20	P3L3	
<b>6</b>	P1L5	P4L28	
<b>7</b>	P2L10	P4L32	
<b>8</b>	P2L11	P4L9	
<b>9</b>	P2L12	P6L12	
<b>10</b>	P2L13	P6L6	
<b>11</b>	P2L17		
<b>12</b>	P4L1		
<b>13</b>	P4L10		
<b>14</b>	P4L11		
<b>15</b>	P4L25		
<b>16</b>	P4L26		
<b>17</b>	P4L27		
<b>18</b>	P4L5		
<b>19</b>	P6L14		
<b>20</b>	P6L15		
<b>21</b>	P6L17		
<b>22</b>	P6L19		
<b>23</b>	P6L3		
<b>24</b>	P6L4		
<b>Mean of design performance</b>	1.202853	1.336327	1.349628
<b>STD of design performance</b>	0.356015	0.636751	0.587594

or waste time which takes more than 300s. For this reason, we omit the time gaps, which are more than 300s.

Additionally, we observe that most of the designers use the time gaps between this range. In order to identify the appropriate distribution that fit these time gaps, we use Kolmogorov–Smirnov test [43] where we check different distribution including Normal, Exponential, Gamma, Pareto, Generalized extreme value (GEV) distribution, Weibull distribution. The test indicates that all of the designers' time gaps follow GEV distribution. Figure 7 shows designer P3L3's empirical time gaps and the corresponding fitted distribution. From the distribution, we identify three parameters which include shape, location, and scale. With these three parameters from 39 designers, we obtain a  $3 \times 39$  embedding matrix. This matrix represents the designers' reflective thinking. After applying the X-mean cluster, we obtain four clusters. Figure 8 shows the four clusters. The result of the clustering is shown in table 5. The result shows that cluster 1 contains nine designers with a mean design performance of 1.1986 and a standard deviation of 0.32. Cluster 2 contains 22 designers with means of 1.31 and a standard deviation of .55, while cluster 3 contains seven designers with a mean design performance of 1.14 and a standard deviation of 0.39. Cluster 4 has only one designer with a performance of 1.10.



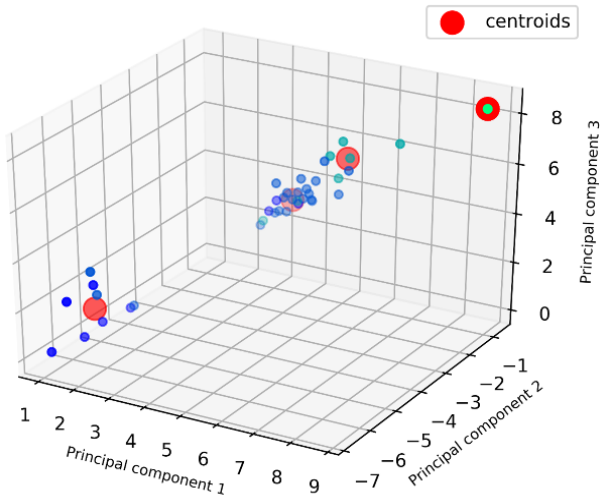


**FIGURE 7: TIME GAP DISTRIBUTION OF DESIGNER P3L3**

However, the Anova test indicates that the difference in design performance among the clusters is not significant (p-value is 0.83).

## 5.2 Discussion

This study aims to understand design thinking from different behavioral perspectives by characterizing them through design embedding. After obtaining the embedding, we apply X-mean cluster on each of the embedding matrices to group the designers. The clustering result indicates that the designers are clustered not according to their design performance, instead of their behavioral patterns. For example, in the aggregated action behavior embedding clustering, the average number of used design process stages are different in each of the clusters. Designers of cluster 3 use a high number of *Synthesis* on average compared to the other clusters' designers. Cluster 3 uses on average 500 *Synthesis*, while cluster 1 and cluster 2 use on average 150 and 233 *Synthesis*, respectively in their design task.

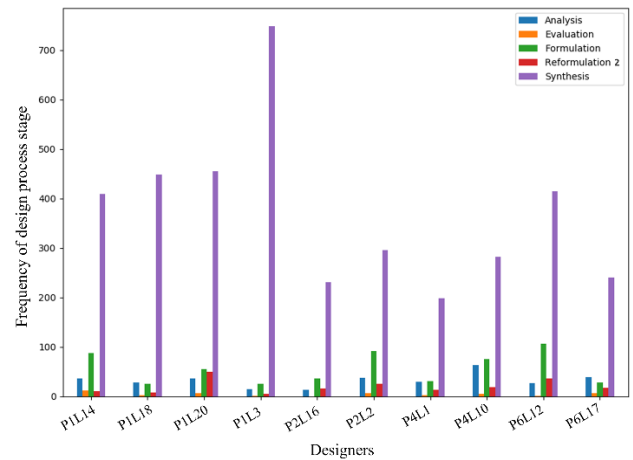


**FIGURE 8: CLUSTER OBTAINED FROM REFLECTIVE THINKING BEHAVIOR**

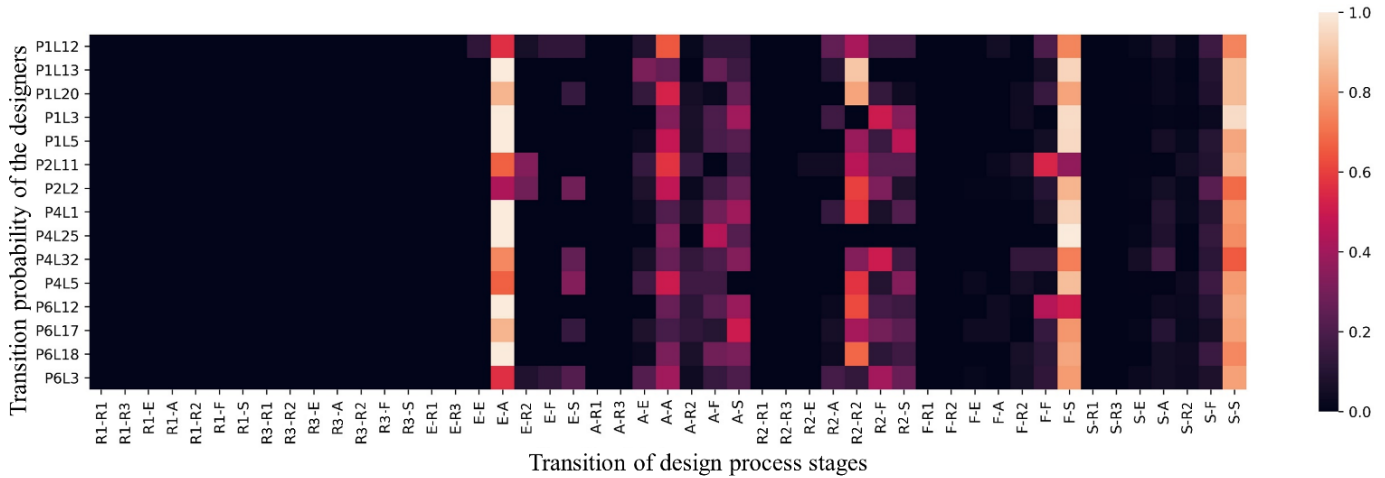
This indicates that designers of cluster 3 are involved in editing design components more frequently than the other designers during the design process. Additionally, we observe a higher number of usage of *Formulation* among the designers of cluster 3 than the others. The average number of the *Formulation* used by cluster 3 is 62, while cluster 1 and cluster 2 use 35 and 40, respectively. Figure 9 shows the design process stage preference of cluster 3.

For reflective thinking behavior embedding clustering, designers in each of the clusters also follow specific design thinking patterns. For example, the designers of cluster 1 use a large number of 1s-3s time gaps compare to the other time gaps. This phenomenon indicates that designers of this group tend to use design actions without much thinking. This behavior also indicates that these designers like to do trial and error more often during the design task. In cluster 2 and cluster 3, designers also follow the same distribution of time gaps. However, unlike cluster 1, designers of the cluster 2 and 3 use relatively low number of 1-3s time gaps. Rather in these clusters, 4-10s time gaps are relatively higher than the designers of cluster 1, which indicates that before using some of the design actions, these designers think. There is only one designer in cluster 4, and his design thinking different from the other designers.

Design embedding of one-step sequential behavior identified from the first-order Markov chain indicates that designers follow several design patterns. By clustering the one-step sequential design embedding, we identified two clusters. In both of the clusters, designers use *Synthesis*→*Synthesis* and *Formulation*→*Synthesis* very frequently. *Synthesis*→*Synthesis* indicates that designers sequentially change the parameters of the design components to achieve the final objective of the design task. For example, after changing the solar panel's tilt angle, designers may change the azimuth of it. *Formulation*→*Synthesis* indicates that after adding a component, the designer tends to change the component's parameter. For example, after adding solar panels, the designer may change its parameter, such as changing the model or changing the solar panels' base height. There are some design patterns that are



**FIGURE 9: PREFERENCE OF DESIGN PROCESS STAGES OF CLUSTER 3**



**FIGURE 10: HEAT MAP OF THE TRANSITION PROBABILITY OF THE DESIGN PATTERNS OF CLUSTER 2**

distinct from each of the clusters. For example, designers of cluster 2 use *Evaluation*→*Analysis* design patterns during their design task, while this pattern is used very rarely among the designers in cluster 1. This pattern indicates that after doing cost evaluation (compare the current cost with the given budget) of the solar panel designer analysis, the solar panels' electricity. Figure 10 shows a heat map of the transition probability of the design patterns found by the designers of cluster 2. The bright square indicates a high transition probability of the corresponding design patterns, where the dark square indicates no or very low transition probability.

## 6. CONCLUSION

In this study, we develop a method to represent design thinking by characterizing behavior from multiple dimensions. We identified five different design behaviors, including design action preference, one-step sequential behavior, contextual behavior, long-term sequential behavior, and reflective design behavior. The design behaviors are characterized by different machine learning and statistical methods, and the design thinking is represented through a latent representation referred to as design embedding. We use the distribution of action preference to characterize action behavior. The First-order Markov model is utilized for one-step sequential behavior. To identify contextual behavior, we use Doc2Vec. A bi-directional LSTM autoencoder characterizes long-term sequential behavior. Finally, we use time gap distribution for reflective design behavior. After identifying the design embedding from each design behavior, we use the X-mean cluster on each embedding to identify similar behaviors. The result indicates that for different embedding, designers are grouped in different clusters. Also, we find that the designers are grouped not actually based on their design performance, rather based on their design behavioral patterns. For example, some designers use a trial and error strategy without much thinking, while others think a bit amount of time before executing design actions. Additionally, we identify several design patterns (i.e., *Synthesis*→*Synthesis* and *Formulation*→*Synthesis*).

The major contribution of this paper is the identification of latent representation (i.e., design embedding) of design thinking through design behaviors from multiple dimensions. The correct implementation of design embedding can be useful in design research in different ways. For example, design embedding can be used to identify designers of similar behavior and identify their design strategies and patterns which can be used to form an efficient design team. Also, in predicting future design decisions, the design embedding can be used in the first place because of its low dimensional space. Furthermore, as the design process is a combination of different design behaviors, all of these design embeddings can be used to develop a predictive model for design performance. In the future study, we will focus on developing a computational model from these embedding which can predict design performance based on their design actions. Particularly, we will develop a regression model from the identified design embedding and correlate with the design performance.

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